|  |  |  |  |
| --- | --- | --- | --- |
| Name: | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ | Date: *\_\_\_\_\_\_* | |
|  | **Subject: Mathematics Specialist**  **Investigation 3- 2018**  **Topic: Fibonacci using matrices**  35 mins | 23  = % | |
| **Weighting:** | *4% of the year.* | |  |
| **Equipment:** | *Curriculum Council Formula sheets, Calculators* | | |
| **Important Information:**  *This in-class validation will be completed under test conditions.*  ***Answers should be rounded appropriately****. All working should be shown in the space provided. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks.*  *No pen, pencils, highlights etc. may be used during reading time. This time is to be used to read through the assessment and check that you understand what is being asked of you. You may speak with the teacher/supervisor during this time (by putting up your hand and waiting patiently for them to approach you) but you may only ask clarification questions and not how to solve the problems. After reading time has ended, you may not ask any more questions.* | | | |

The Fibonacci sequence is the set of numbers 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, …..

The Fibonacci sequence is defined as *Fn+2 = Fn + Fn+1* with the first two terms equal to one, i.e. *F1* = 1 and *F2 =* 1 and where *Fn* stands for the nth Fibonacci number.

**Question 1(3 marks)**

Let *M* = .

It has been conjectured that  i.e. 

Test the rule for *n* = 4.

**Question 2** **(6 marks)**

Given , i.e. *M n* 

(a) Show that the rule works for *n = 3.* (3)

(b) Show that the rule works for *n* = 10. (3)

**Question 3** **(10 marks)**

Given  then it follows that .

(a) Show that . (3)

(b) Test the rule  for *n* = 2 and for *n* = 3. (4)

(c) Assume the rule is true for *n* = “*n*” and prove the validity of the rule for *n* = “*n* +1”.

(3)

**Question 4** **(4 marks)**

Given *M n* = 

and *M n* =

(a) State the formula for *Fn* the *n*th term of the Fibonacci series in terms of . (1)

(b) Show how to evaluate without using a calculator. (3)

**End of questions**

**Part 2: In-class validation**

**Solutions and marking key**

**Question 1**

|  |  |  |
| --- | --- | --- |
| Solution | Marking key/mathematical behaviours | Marks |
| *n* = 4    *∴* the rule works for *n* = 4 | * Substitutes into formula correctly * Multiplies the matrices correctly * Shows the result gives the appropriate Fibonacci numbers | 1  1  1 |

**Question 2 (a)**

|  |  |  |
| --- | --- | --- |
| Solution | Marking key/mathematical behaviours | Marks |
| The rule works for *n* = 3 | * Calculates the matrix correctly * Identifies the correct Fibonacci numbers * Makes an appropriate conclusion | 1  1  1 |

**Question 2 (b)**

|  |  |  |
| --- | --- | --- |
| Solution | Marking key/mathematical behaviours | Marks |
| But  ∴  Therefore the rule works for *n* = 10 | * Calculates correct matrix * Identifies the Fibonacci terms * Makes an appropriate conclusion | 1  1  1 |

**Question 3 (a)**

|  |  |
| --- | --- |
| Solution  (a)    But  ∴ | |
| Marking key/mathematical behaviours | Marks |
| * Calculates the value of each determinant * States equality | 2  1 |

**Question 3 (b)**

|  |  |
| --- | --- |
| Solution | |
| Marking key/mathematical behaviours | Marks |
| * States the realationship for *n = 2* * Checks LHS = RHS * States the realationship for *n = 3* * Checks LHS = RHS | 1  1  1  1 |

**Question 3 (c)**

|  |  |
| --- | --- |
| Solution | |
| Marking key/mathematical behaviours | Marks |
| * Correctly defines the equation for *m* * Substitutes for  *m=n + 1* * Provides relevant conclusion | 1  1  1 |

**Question 4 (a)**

|  |  |
| --- | --- |
| Solution  *Fn =* | |
| Marking key/mathematical behaviours | Marks |
| * Correctly identifies formula for *Fn* | 1 |

**Question 4 (b)**

|  |  |
| --- | --- |
| Solution  (b) | |
| Marking key/mathematical behaviours | Marks |
| * Equates the given expression to * Correctly evaluates using the Fibbonacci sequence * Gives the exact value of the expression | 1  1  1 |